

# APPENDIX: FORMULAE AND RULES

## POINTS AND LINES




<b>MIDPOINT</b>	$\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$
<b>DISTANCE</b>	$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
<b>SLOPE</b>	$\frac{y_2 - y_1}{x_2 - x_1}$
<b>LINE THROUGH (<math>x_1, y_1</math>) WITH SLOPE <math>m</math></b>	$y - y_1 = m(x - x_1)$
<b>PARALLEL LINES</b>	$m_1 = m_2$
<b>PERPENDICULAR LINES</b>	$m_1 m_2 = -1$

## DIFFERENTIATION

<b>SUM AND DIFFERENCE</b>	$\frac{d(u \pm v)}{dx} = \frac{du}{dx} \pm \frac{dv}{dx}$
<b>CONSTANT FACTOR</b>	$\frac{d(ku)}{dx} = k \frac{du}{dx}$
<b>PRODUCT RULE</b>	$\frac{d(uv)}{dx} = u \cdot \frac{dv}{dx} + v \cdot \frac{du}{dx}$
<b>QUOTIENT RULE</b>	$\frac{d(u/v)}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$
<b>CHAIN RULE</b>	$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

<b>INVERSE FUNCTIONS</b>	$\frac{dx}{dy} \cdot \frac{dy}{dx} = 1$	
<b>FUNDAMENTAL THEOREM OF CALCULUS</b>	$\frac{d \int f(x) dx}{dx} = f(x)$	$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$

## STATIONARY POINTS AND OPTIMISATION

Optimising $y = f(x)$ : Put $\frac{dy}{dx} = 0$ .			Optimising $z = f(x, y)$ : Put $\frac{\partial z}{\partial x} = \frac{\partial z}{\partial y} = 0$ .		
<b>FIRST DERIVATIVE TEST</b>			<b>SECOND DERIVATIVE TEST</b>		
 Local MIN	 Local MAX	 Inflection	$y'' < 0$ Local MAX	$y'' = 0$ TEST FAILS	$y'' > 0$ Local MIN

## INTEGRATION

$\int [u \pm v] dx = \int u dx \pm \int v dx$		$\int ku dx = k \int u dx$
IF $\int f(x) dx = F(x)$	$\int_a^b f(x) dx = [F(x)]_a^b = F(b) - F(a)$	$\int f(ax + b) dx = \frac{1}{a} F(ax + b)$
<b>AREA BETWEEN CURVES from <math>a</math> to <math>b</math></b>		$\int_a^b (\text{top } y - \text{bottom } y) dx$

## SPECIAL FUNCTIONS

$y$	$\frac{dy}{dx}$	$\int y \, dx$ (include + c)
$x^n$	$nx^{n-1}$	$\frac{1}{n+1} x^{n+1}$ if $n \neq -1$
$\frac{1}{x}$	$-\frac{1}{x^2}$	$\log x$
$e^x$	$e^x$	$e^x$
$\log x$	$\frac{1}{x}$	$x \log x - x$
$\sin x$	$\cos x$	$-\cos x$
$\cos x$	$-\sin x$	$\sin x$
$\tan x$	$\sec^2 x$	$\log(\sec x)$

## NUMERICAL METHODS

<b>NEWTON'S METHOD</b> to solve $y = f(x) = 0$	$x_1 = x_0 - \frac{y_0}{y_0'}$
$\int_a^b y \, dx$ using $n$ strips of width $h$	
<b>TRAPEZIUM RULE</b>	$\frac{h}{2}$ [First + Last + 2(Sum of others)].
<b>CUBIC FIT RULE</b>	TRAPEZIUM RULE - $\frac{h^2}{12} [y']_a^b$
<b>SIMPSON'S RULE</b> even number of strips	$\frac{h}{3}$ [first + last + 2(sum of other evens) + 4(sum of odds)]

## TANGENTS

<b>Tangent to <math>y = f(x)</math> at <math>x = a</math></b>	$y - f(a) = f'(a)(x - a)$
<b>Point of contact of tangent(s) to <math>y = f(x)</math> through <math>(a, b)</math></b>	Solve $b - f(t) = f'(t)(a - t)$ Point of contact is $(t, f(t))$

## LINEAR DIFFERENTIAL EQUATIONS

$\frac{dy}{dx} = ky$	$y = Ae^{kx}$
$a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$ where $\lambda, \mu$ are two real zeros of $aX^2 + bX + c$ .	$y = Ae^{\lambda x} + Be^{\mu x}$

## LAGRANGE MULTIPLIERS

<b>PARTIAL DERIVATIVES</b>	$\frac{\partial z}{\partial x}$ = derivative keeping all other variables fixed
<b>MAX and MIN of <math>F(x, y)</math> subject to the constraint <math>\varphi(x, y) = 0</math>.</b>	Solve $\frac{\partial z}{\partial x} = \frac{\partial z}{\partial y} = \varphi(x, y) = 0$ .

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## LIST OF TITLES

### GENERAL

- The Mathematics At The Edge Of The Rational Universe

### ELEMENTARY

- Basic Mathematics
- Concepts of Algebra
- Concepts of Calculus

### 1<sup>st</sup> YEAR UNI

- Techniques of Algebra
- Techniques of Calculus
- Matrices

### 2<sup>nd</sup> YEAR UNI

- Linear Algebra
- Languages & Machines
- Discrete Mathematics

### 3<sup>rd</sup> YEAR UNI

- Group Theory vol 1
- Group Theory vol 2
- Galois Theory
- Geometry vol 1
- Geometry vol 2
- Topology
- Set Theory
- Number Theory
- Graph Theory
- Complex Variables

### POSTGRADUATE

- Ring Theory
- Representation Theory
- Quadratic Forms
- Group Tables vol 1
- Group Tables vol 2

